RESONANT LASER COOLING of RELATIVISTIC CHARGES BEAMS

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Abstract

In work is considered average transverse dynamics of an electron beam in the autoresonant laser. It is shown, that in approach of the given external electromagnetic wave (small gain free electron laser) transverse emittance of a beam of the charged particles decreases. This for the first time found out effect can be used for cooling beams of various accelerators of the charged particles. In the field of particle energy about 100 in the mass energy units, beam energy losses are negligible. This method of charge beams cooling may be applied to electron, muon and various heavy particle beams.

Enough methods of the charged particle accelerator beam cooling based on tested and used on various accelerators physical principles now are known: radiating [1], electron [2], stochastic [3] and Doppler [4] cooling. However they by the opportunities do not meet more modern requirements. Necessity for improvement of various parameters of these methods has resulted last years in consideration of new beam cooling methods from among which it is necessary to note autoresonant [5], undulator [6], ionization [7], laser [8] and stimulated radiation or Doppler [9] cooling. The special attention is given to cooling by backward Compton scattering of free electrons on photons [10-12]. It is known, however, that the stimulated scattering under other equal conditions has greater cross-section [9,13], than those of free particles. Therefore, as against the above-stated ways of cooling, in the given work the opportunity of transverse cooling of a beam of relativistic charges is considered at the stimulated or resonant interaction of particles with a flat electromagnetic wave (photons). For creation of conditions of resonant bunch - wave interaction is entered the homogeneous magnetic field directed along a direction of movement of a beam. The magnetic field creates density correlations of a beam current on the laser wavelength that increases bunch - wave interaction and makes it selective. Interaction moving on a periodic trajectory electrons with a flat monochromatic electromagnetic wave is one of possible ways of amplification of an electromagnetic wave in the free electron laser. Offered in [14] autoresonant free electron laser, based on interaction of the charged particle with the electromagnetic wave extending along a homogeneous magnetic field $B_0 \parallel z$, is characterized by a detuning constancy from an exact resonance at charged particle energy change. In [15] shown, that the detuning constancy connected to existence of motion integral

$$I = \gamma - P_z$$

takes place only in approach of the given external wave. Here γ is the full energy, and P momentum of a particle along a magnetic field (in terms of m=c=1). In the present work influence of autoresonant interaction of a particle with a wave on a beam transverse emittance is examined. Taking into account connection $\gamma = \sqrt{P_z^2 + P_\perp^2 + 1}$, from (1) it is easy to receive for a transverse kinematic momentum of a particle

$$P_\perp^2 = 2I\gamma - 1 - I^2$$

Differentiating this expression on time and averaging on all particles of a beam, we shall receive the equation for change of an average square of a cross-kinematic momentum of a beam

$$\frac{d}{dt}\langle P_\perp^2 \rangle = 2\langle I\dot{\gamma} \rangle$$

As shown in works [14-16], trajectories of electrons are screw lines with radius varying along a way. As drift of the center of a Larmor circle absence and I ¿ 0, interaction in a mode of the laser ($\dot{\gamma} < 0$) means reduction of a beam transverse emittance, because of negativity of the right part in (2). In a laser mode the amplified wave carried away with itself a part of beam energy. It is very important to find what part of wave carried energy is from full beam energy and what part is from beam emittance, which means beam cooling (emittance reduction). As it is not difficult to receive from (1), relative speed $\dot{P}_z/P_z$ of a longitudinal momentum change $2P_\perp^2/(1+P_\perp^2)$ times less than relative speed $\dot{P}_\perp/P_\perp$ of a cross-kinematic momentum change. Therefore emittance reduction will be effective for beams with $\langle P_\perp^2\rangle \ll 1$, when full energy losses are small. But in practical applications it is possible also to compensate beam energy losses by external accelerating elements. In approach of infinitesimal changes of a cross-kinematic momentum at interaction with a wave, when motion in a cross plane is determined by a magnetic field B_0, for $\dot{\gamma}$ in a field of the circular-polarized wave with frequency (and amplitude it is easy to receive

$$\dot{\gamma} = \xi\omega\frac{P_\perp}{\gamma}\cos\varphi$$

where $\xi = {}^{eE}\!/_{\omega}$ is the dimensionless amplitude of a wave electric field, and (is a phase of the Larmor rotations, counted from a wave phase in a point of a presence of a particle. In used by us approach it is easy to receive the equation for change of an average longitudinal momentum or energy of a beam

$$\langle\dot{p}\rangle = \langle\dot{\gamma}\rangle = \left\langle \xi\omega\frac{p_\perp}{\gamma}\cos\varphi \right\rangle$$

Averaging on all particles of a beam in the equation (2) allows instead of a detailed trajectory of one particle in the given external fields to investigate average on a beam of size, using the kinetic approach. At presence of an electromagnetic wave with $\vec{E},\vec{B} \sim e^{i(\omega t - kz)}$ which we shall count small in comparison by a magnetic field $\vec{B}_0$, in a beam there are correlations of density which result in modulation of a beam on (and accordingly, to a nonzero right part (2). For calculation of this effect we shall present function of distribution of a beam as [17] $f = f_0 + \delta f$, where f_0 is the basic equilibrium function of distribution, and $|\delta f| \ll f_0$ is the small amendment to f_0, cause by a wave. Near to a resonance it is convenient to present the amendment as Fourier series from a phase $\varphi : \delta f = \sum g_s(P_\perp, P_z)e^{is\varphi}$.

Substituting in the kinetic equation and neglecting members of the second order, we shall receive the following decision for factors

$$g_s = \frac{Q_s}{i(s+\alpha)}; Q_s = \frac{1}{2\pi}\int\limits_0^{2\pi} d\tau e^{-is\tau}Q(P_z, P_\perp, \tau); Q = \frac{e}{\omega_B}\frac{\partial f_0}{\partial \vec{P}}(\vec{E} + \frac{1}{\omega}[\vec{\ldots}$$

where $\alpha = {}^{(kv_z - \omega)}\!/_{\omega_B}$, $\omega_B = {}^{eB_0}\!/_{\gamma}$ is the Larmore frequency,τ is the phase of integration and used $\omega\vec{B} = [\vec{k}\vec{E}]$ connection for a field of a wave. Near to a simple cyclotron resonance the basic member in Four-decomposition will be a member with s=1. For Gaussian distribution function on a cross momentum

$$f_0 = \frac{1}{\pi\langle P_\perp^2\rangle}e^{-P_\perp^2/\langle P_\perp^2\rangle}\delta(P_z - P_0)$$

with the account $I \approx (1+P_\perp^2)/2P_0$ in a relativistic case $P_0 \gg 1 + P_\perp^2$ we shall receive

$$\delta f = -\xi \frac{P_\perp(1+P_\perp^2)}{\langle P_\perp^2\rangle i(\Delta_\parallel - P_\perp^2)} e^{i\varphi} f_0$$

where $\Delta_\parallel = 2\frac{\Omega}{\omega}P_0 - 1; \Omega = \omega_B\gamma$. Approximation of a longitudinal - monochromatic beam in the f_0 as it is easy to estimate from resonant term $(1+\alpha)^{-1}$, is fair, if the relative spread on a longitudinal momentum $(P_z - P_o)/P_0$ is much less $\langle P_\perp^2 \rangle$. At averaging on f there is a usual pole in such cases in a point $p_\perp^2 = \Delta_\parallel$, caused by a resonant multiplier. By a rule of detour of Landau poles (replacement $\omega \to \omega + i0$) we receive for integral interesting us [17]:

$$\int_0^\infty \frac{f(z)}{\Delta_\parallel - z} dz = \int \frac{f(z)}{\Delta_\parallel - z} dz + i\pi\delta(\Delta_\parallel - z)$$

where $z = p_\perp^2$, and the right integral is meant in sense of a principal value. As the real part of the equation (2), has physical sense only at $\Delta_\parallel \leq 0$ if the right part only imaginary, speed of change of a cross momentum equal to zero. And we receive:

$$\frac{d}{dt}\left\langle p_\perp^2 \right\rangle = -\xi \frac{\pi\omega}{2p_0^2 \left\langle p_\perp^2 \right\rangle} \Delta_\parallel (1+\Delta_\parallel) e^{-\Delta_\parallel / \langle p_\perp^2 \rangle}$$

Maximum of the right part in a case $\left\langle p_\perp^2 \right\rangle \ll 1$ ($p_0 \approx const$) is achieved in a point $\Delta_\parallel = \left\langle p_\perp^2 \right\rangle$, and its solution is

$$\left\langle p_\perp^2 \right\rangle = (\left\langle p_{\perp 0}^2 \right\rangle^2 - \frac{2\pi^2}{e}\xi^2 \frac{z}{2\lambda p_0^2})^{1/2}$$

where $P_{\perp 0}$ is the initial value of an average square of a cross momentum of a beam, (λ is a length of a wave, and z =ct is a length of a beam way.

If $\Delta_\parallel = const$ and much less $\left\langle p_\perp^2 \right\rangle$, we shall receive:

$$\left\langle p_\perp^2 \right\rangle = (\left\langle p_{\perp 0}^2 \right\rangle^3 - \xi^2 \frac{6\pi^2 z}{2\lambda p_0^2} \Delta_\parallel)^{1/3}$$

Decreasing of a beam cross emittance at autoresonant bunch - wave interaction must be significant as well for a bunch modulated on ϕ which function of distribution we shall choose in such kind:

$$f(p_\perp, p_z, \varphi) = f_0(1 - \varepsilon\cos\varphi)$$

where $0 < \varepsilon \leq 1$ is the depth of modulation, and f_0 is defined from (5). After averaging (2) on this function we shall receive:

$$\frac{d}{dt}\langle p_\perp \rangle = -\varepsilon\xi \frac{\omega}{2p_0^2}(1 + \frac{6}{\pi}\langle p_\perp \rangle^2),$$

where $\langle p_\perp \rangle = \frac{\sqrt{\pi}}{2}\sqrt{\langle p_\perp^2 \rangle}$ connection is taken into account. From here at $\left\langle p_\perp^2 \right\rangle \ll 1$ ($p_0 \approx const$), we shall receive

$$\langle p_\perp \rangle = p_{\perp 0} - \varepsilon\xi\pi \frac{z}{2\lambda p_0^2}$$

The condition of small changes of the cross momentum, used at a conclusion of all formulas, means actually restriction on passed way length:

$$1 \leq \frac{z}{2\lambda p_0^2} \ll A$$

where $A = \frac{e\left\langle p_{\perp 0}^2 \right\rangle^2}{2\pi^2\xi^2}; \frac{\left\langle p_{\perp 0}^2 \right\rangle^3}{2\pi^2\xi^2\Delta_\parallel}; \frac{p_{\perp 0}}{\varepsilon\xi\pi}$ for formulas (8), (9) and (12) respectively. And the left inequality is consequence to approach of adiabatic switching of a wave field. As in our case the field is switched instantly the approach adiabatic slow engaging of a wave field is fair at times greater than relaxation time. That is expression (6) for fair if the beam has passed a way greater than $2p_0^2\lambda$. We shall emphasize that the condition (13) follows from a method of calculation. The equation (2) fairly in approach of the given external field when Leangmure frequency of a beam is much less than frequency of the laser [15]. It specifies connection between speed of transverse momentum change and speed of a beam energy change. For a beam with$P_0 = 100$ and divergence $\vartheta = 10^{-3}$ the laser with $\xi = 10^{-2}$ the length of a way on which transverse emittance change to 100 makes 80 cm, thus length of a relaxation about 20cm. Thus, autoresonant cooling of beams of relativistic charges is much effective and faster than known and listed above methods of cooling. It is very effective for beams with the Lorenz-factor about 100, but is applicable also to GeV particles and for achievement comparable to [10, 11] rates of cooling sources of electromagnetic radiation, in this case masers, with rather low and more real capacities are required. We shall notice that intensity of cooling depends on parameter ξ, which is easy for making about unity in long-wave sources of radiation, for example in masers. Thus it is much less than loss of energy of a beam. If to add to this an opportunity of application of the longitudinal and -or cross-section non-uniform magnetic field that enables scanning in case of a beam with the big divergence, and also selective cooling advantage of this method becomes obvious. Hence, autoresonant cooling, that is laser cooling of beams of charges in a longitudinal magnetic field has a number of advantages even in comparison with the newest laser [10, 11] and undulator cooling methods, as because of a dependence of cooling speed from a cross momentum of particles and a beam as a whole, and in connection with formation of correlations of currents of a beam and ample opportunities of their management by heterogeneity of a magnetic field for increase of efficiency and selectivity of cooling. In this sense it is comparable to the stimulated cooling methods. This cooling, if applied, can to increase significantly the luminosity of the storage ring and synchrotron beams, due to decrease of the beam sizes. Under certain conditions autoresonant cooling can succesfully applied to linac beams too and give a possibility to achieve highest density and luminosity beams with small sizes and momentum spread.

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